SM358

Tutor-Marked Assignment 01

Contents **Cut-off date**

Please send all your answers to the tutor-marked assignment (TMA) to reach your tutor by 12 noon (UK local time) on or before the cut-off date shown on the SM358 website. Your TMAs should be submitted through the eTMA system unless there are difficulties which prevent you from doing so. In these circumstances, you must negotiate with your tutor to get their agreement to submit your assignment on paper. The eTMA system allows for eTMA submission directly to the university 24 hours a day, and either gives you confirmation that your eTMA has been submitted successfully or, if there has been a problem, an error message informing you of the problem and what steps you can take to overcome it. If you submit online you must keep your receipt code in case you need to prove successful submission.

General information about policy and procedure is in the *Assessment Handbook* which you can access from your StudentHome page. However, there are a number of ways in which SM358 eTMA submission differs from what is described there. These are described in the document *Producing eTMAs for Level 3 physics and astronomy modules* on the SM358 website. See also the *SM358 Introduction and Guide* for module-specific information.

Of particular importance is the test submission, TMA 00. This will enable you to familiarize yourself with the system and allow your tutor to check that the format in which you save your TMAs is compatible with their own computer software. It is your responsibility to make sure that you submit documents in a compatible format and we strongly recommend that you submit TMA 00. TMAs submitted in an incorrect format may not be marked.

If you are submitting a paper copy, please allow sufficient time in the post for the assignment to reach its destination on or before the cut-off date. We strongly advise you to use first-class post and to ask for proof of postage. Do not use recorded delivery or registered post as your tutor may not be in to receive it. Keep a copy of the assignment in case it goes astray in the post. You should also include an appropriately completed assignment form (PT3). You will find instructions on how to fill in the PT3 form in the *Assessment Handbook*. Remember to fill in the correct assignment number (01).

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5.1 PT/TeX

This first booklet provides general advice on answering TMA questions as well as the questions for TMA 01. Although the marks for your assignments do not count directly towards your SM358 result, they are an essential part of your learning and you are required to engage satisfactorily with them. Please refer to the *SM358 Introduction and Guide* for additional information about the module assessment.

Assignment cut-off dates
The cut-off dates for the assignments provide an important element of pacing for your study of SM358 and they are spread fairly uniformly through the year, leading up to the exam. **You should regard these dates as fixed points**. *Any extension to a TMA cut-off date requires prior permission from your tutor, which may not always be given. Extensions may be granted in exceptional circumstances but it will never be possible to have an extension of more than 3 weeks.* Your tutor will, of course, be willing to discuss with you the best strategies for catching up if you have fallen behind, and should be able to help with questions if you are stuck.

General advice on answering SM358 assignment questions

The TMA questions allow you to demonstrate that you have achieved particular learning outcomes for the module. These learning outcomes are listed in Section 6.5 of the *SM358 Introduction and Guide*. They include knowledge and understanding of the module content, the ability to apply this knowledge and understanding to the solution of quantum-mechanical problems, the ability to explain concepts, phenomena and applications of quantum mechanics, and the ability to communicate effectively your solutions and explanations. In each TMA booklet we indicate which of the learning outcomes are assessed in the assignment, and for each question we indicate the main achievements (from the lists at the end of each chapter) that are assessed.

When marking and commenting on your TMA answers, your tutor will be assessing the extent to which you have achieved the learning outcomes. A proportion of the marks will be awarded for getting to the correct answer, but proportions will also be awarded for explaining your reasoning, for well-structured answers (both for problems and discursive answers), for correct use of terminology and notation, and so on.

The three *Tutorial problem* videos on the DVDs provide a wealth of advice on tackling assignment questions, and you are strongly encouraged to view the first video before doing TMA 01 and the second and third videos before doing TMA 02. In addition, we summarize below a number of general points that you should have in mind when answering the TMA questions. Many of these are likely to be familiar from previous modules, but they bear repetition at the start of a new module.

1. **If you get stuck** Worked examples and exercises in the books, and iCMA questions and additional exercises on the website, often provide clues. If you are unsure how to get started, do not hesitate to contact your tutor who will be more than happy to point you in the right direction. One of the advantages of formative assessment is that you can ask for help before submitting your work, and this can be freely given. Do not ignore or omit any question you find hard. If you do, you will miss out on a valuable learning opportunity, and may find yourself at a disadvantage in the final exam.
2. **Define the terms used** Good scientific communication requires clarity about the meaning of terms and symbols. You will be expected to state the meaning of scientific terms that you use in your answers if they are not introduced in the question.
3. **Appropriate use of notation** A great deal of meaning is conveyed by notation. You have freedom to choose symbols for a given situation, but should avoid potentially misleading choices. For example, wave functions are usually denoted by upper case Greek letters, such as Ψ or Φ, while eigenfunctions are denoted by lower case Greek letters, such as ψ or ϕ. The operator corresponding to a quantity O is generally written as $\widehat{\mathrm{O}}$, with a hat. It would be confusing if you departed from these conventions. The notation for ordinary vectors also needs some care. In handwritten work, each vector is indicated by a wavy underline $\underset{\sim}{r}$; without underlining, symbols such as r or p refer to the

magnitudes of the vectors **r** and **p**. Remember also that if one side of an equation is a vector quantity, then the other side must be a vector too.

4. **Laying out answers to problems** Effective communication with your tutor (or anyone else) is helped by a neat and logical layout of your answers. Perhaps the best advice is for you to look carefully at the layout of the solutions to worked examples and exercises in the SM358 books. These solutions also provide examples of the appropriate level of explanation required in your TMA answers. Note that, although you may be able to solve a particular problem by writing out a few lines of algebra that lead to the correct answer, you will only be awarded full marks if you provide explanation and justification at appropriate stages in your answer.
5. **Answers requiring written explanation or description** Some of the TMA questions allow you to demonstrate and develop your scientific writing skills. Your tutor will be assessing whether your answer is coherent (i.e. topics and argument presented in a logical order and clearly linked to the question), clear (i.e. the meaning is unambiguous, and there is correct use of English and appropriate equations), and concise. Guidance on the length of written answers will generally be given. If you are tempted to write much more than the guideline indicates, it may be because you have not written a clear and concise answer, or you may have included material that is irrelevant or secondary. Conversely, if your answer is much shorter than the guideline indicates, then you should consider whether you have omitted relevant information, or written so tersely as to make the answer difficult to follow.
6. **Include equations and diagrams where helpful** When answering a discursive question, do not hesitate to quote relevant equations. In physics, equations are often the most precise and succinct way of presenting information. Remember to define in words the meanings of any symbols that may not be obvious. Hand-sketched diagrams can also help to clarify an answer, and can save many words of description. Simple pencil-drawn line diagrams are all that is generally required — neat, but not necessarily artistic. If submitting electronically, you may wish to scan in a hand-drawn sketch.
7. **Units** The values of most physical quantities in quantum mechanics have units as well as a numerical value. For a value of energy E, it would be correct to write '$E = 2.5\,\mathrm{J}$', but it certainly would not be acceptable to write '$E = 2.5$'. Also, it is not correct to write 'E joules', since we assume that a symbol such as E, representing an energy, already contains a unit for energy. Mathematicians sometimes take liberties about how they handle units, but scientists generally treat units much more seriously. Answers to problems should *always* include the appropriate unit, and when writing out equations that include numerical values, the units on each side of the equation must always balance. The solutions to the exercises in Book 1 provide many examples of good practice.
8. **Significant figures** Numerical answers should always be quoted to the appropriate number of significant figures. In SM358 we often quote values for the quantities used in numerical calculations to two significant figures, and we provide values of physical constants to three significant figures. In such cases, the answers should be quoted to two significant figures. For intermediate steps in a calculation it is good practice to record *one more* figure than is significant, to avoid the accumulation of rounding errors in the intermediate steps, leading to an error in the final answer.
9. **Checking answers** We all make slips in algebraic derivations and in evaluating numerical results, so it is advisable to check your answers. There are various ways to do this, depending on the problem. These include checking that units balance in derived equations, checking numerical results with rough estimates obtained by rounding all of the numbers in the calculation, looking at limiting cases for algebraic expressions and asking yourself whether the limiting behaviour is reasonable, checking whether an expression increases or decreases in an appropriate way when the value of a parameter is changed, and so on. The tutorials on the DVD include examples of these techniques.
10. **Plagiarism** You are encouraged to discuss the SM358 materials and assignment questions with other students, but the answers to the assignment questions must be your own work. This does not stop you making judicious use of material from other sources, but you must acknowledge such use by giving the author's name, the year of publication,

the name of the publication in which it appears (or the website address), and the edition or volume number and the page number. However, there is no need to give references for standard equations in the SM358 texts. You should read the University's guidelines on plagiarism in the *Assessment Handbook*, available online from your StudentHome page. To check that all students are working in a fair and academically appropriate manner, the Open University is currently using some text-comparison software to detect potential cases of plagiarism in work that is submitted for assessment. Details of how this is implemented in this module are given on the SM358 website.

This assignment is related to the first four chapters of Book 1. Material in the remaining chapters of Book 1 will be assessed in TMA 02. Your answers for this assignment will provide evidence of your achievement of many of the learning outcomes, as listed in the *SM358 Introduction and Guide*, Section 6.5. In particular, this assignment tests all the *Knowledge and understanding* learning outcomes, skills 1, 2, 3 and 5 of the *Cognitive skills* outcomes and all of the *Key skills* outcomes.

Question 1

This question carries 20% of the marks for this assignment. It relates mainly to Chapter 3 of Book 1, and particularly to Achievements 1.3 and 3.5.

(a) An F-centre can be modelled as an electron trapped in a cubic three-dimensional infinite square well. Calculate the wavelength of the electromagnetic radiation emitted when an electron makes a transition from the third energy level, E_3, to the lowest energy level, E_1, in such a well. Take the sides of the cubic box to be of length $L = 3.2 \times 10^{-10}\,\mathrm{m}$ and the electron mass to be $m_\mathrm{e} = 9.11 \times 10^{-31}\,\mathrm{kg}$. (16 *marks*)

(b) Specify the degrees of degeneracy of the E_1 and E_3 energy levels, explaining your reasoning. (You may ignore any effects due to the spin of the electron; spin is not discussed until Book 2 of this module.) (4 *marks*)

In Question 2 you may find the following results useful:

$$\int_{-a}^{a} f(x)\,\mathrm{d}x = 0 \quad \text{if } f(x) \text{ is an odd function,}$$

$$\int_{-\infty}^{\infty} \mathrm{e}^{-x^2}\,\mathrm{d}x = \sqrt{\pi},$$

$$\int_{-\infty}^{\infty} \mathrm{e}^{-x^2}\mathrm{e}^{-\mathrm{i}bx}\,\mathrm{d}x = \sqrt{\pi}\,\mathrm{e}^{-b^2/4} \quad \text{where } b \text{ is a real constant.}$$

Question 2

This question carries 40% of the marks for this assignment. It relates mainly to Chapters 2 and 4 of Book 1, and particularly to Achievements 2.3, 2.6, 4.3, 4.5 and 4.6.

At time $t = 0$ a particle is described by the one-dimensional wave function

$$\begin{aligned}\Psi(x,0) &= \left(\frac{2\alpha}{\pi}\right)^{1/4} \mathrm{e}^{-\mathrm{i}kx}\,\mathrm{e}^{-\alpha x^2} \\ &\equiv \left(\frac{2\alpha}{\pi}\right)^{1/4} \mathrm{e}^{-\mathrm{i}kx-\alpha x^2}, \qquad (1)\end{aligned}$$

where k and α are real positive constants.

(a) State Born's rule in the context of one-dimensional wave mechanics and explain why this rule leads to the requirement for wave functions to be normalized. Verify that the wave function $\Psi(x,0)$ in Equation 1 is normalized. (You may need to use a standard integral, after changing the variable of integration appropriately.) (13 *marks*)

(b) Write down the sandwich integral rule for the expectation value of momentum. Hence find the expectation value of the momentum, $\langle p_x \rangle$, in the state described by $\Psi(x,0)$. (13 *marks*)

(c) Given that $\langle p_x^2 \rangle = \hbar^2(k^2 + \alpha)$ in the state described by $\Psi(x, 0)$, what is the uncertainty of the momentum, Δp_x, in this state? (6 *marks*)

(d) Suppose that the particle is in a potential energy well with the normalized ground-state energy eigenfunction

$$\psi_0(x) = \left(\frac{2\alpha}{\pi}\right)^{1/4} \mathrm{e}^{-\alpha x^2},$$

and corresponding energy eigenvalue E_0. With the particle in the state described by Equation 1, use the overlap rule to find the probability that a measurement of the particle's energy at time $t = 0$ will give the ground-state energy, E_0. (Your answer should be a function of α and k.) (8 *marks*)

Question 3

This question carries 40% of the marks for this assignment. It relates to Chapters 1, 2 and 4 of Book 1, and particularly to Achievements 1.2, 1.4, 1.8, 2.4, 2.5, 2.6, 4.2, 4.3, 4.4 and 4.7.

In Newtonian mechanics, the state of an isolated system is represented by giving the positions and momenta of all the particles in the system. If these are known at a given instant, Newton's laws tell us how the state of the system will evolve in time, allowing us to predict precisely the results of measurements made on the system at a later time.

This question asks you to provide an overview of the contrasting situation that applies in quantum mechanics, describing **how predictions are made in an uncertain quantum world**. Structure your answer by addressing, in order, the following issues:

(a) In what sense is quantum mechanics fundamentally indeterministic? Give two physical examples of such indeterminacy in real systems. (5 *marks*)

(b) Describe the role of the Heisenberg uncertainty principle in undermining the Newtonian description of the state of a system. (4 *marks*)

(c) Outline the role of the wave function in describing the state of an isolated system in one-dimensional wave mechanics. Is the description given by the wave function as complete as can be? Discuss whether the indeterminacy of quantum mechanics arises from our lack of knowledge of the time-evolution of the wave function in between measurements (as described by Schrödinger's equation), or whether it has another origin. (8 *marks*)

(d) Describe the role of eigenvalue equations in determining which energy values are possible in a bound system. Are there any situations in which quantum mechanics unambiguously predicts the result of an energy measurement with certainty? How does the collapse of the wave function influence the result of an energy measurement taken immediately after another energy measurement? (6 *marks*)

(e) Explain the roles of the overlap and coefficient rules in predicting the probability of obtaining a given value when an energy measurement is made in a given state. (6 *marks*)

(f) Explain the role of Born's rule in predicting the probability of finding a particle within a given interval? How does the collapse of the wave function influence the result of a position measurement taken immediately after another position measurement? (5 *marks*)

(g) Finally give a brief summary of the types of prediction that are possible in quantum mechanics. Comment on the extent to which these predictions can (in principle) be compared with experiment by, for example, making repeated measurements. (6 *marks*)

Your answers should be based on a concise summary of relevant material in Chapters 1–4 of Book 1, suitable for a fellow SM358 student to read as an overview. The total length of all your answers should be about 1000 words (and no more than 1250 words). You are encouraged to include any relevant equations, but may restrict them to cases that involve a single particle in one dimension.

State the number of words you have used at the end of your answer. You will not be penalized for overlong answers but note the advice given in point 5 on page 3 of this assignment booklet. The advice given in point 6 is also relevant to this question.

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